Goodness of Fit Statistics

for Poisson Regression

Outline

- Example 3: Recall of Stressful Events
- Goodness of fit statistics
  - Pearson Chi-Square test
  - Log-Likelihood Ratio test

Example 3: Recall of Stressful Events

- Let us explore another (simple) Poisson model example (no covariate to start with)

Example 3: Recall of Stressful Events

- Participants of a randomised study where asked if they had experienced any stressful events in the last 18 months. If yes, in which month?

- 147 stressful events reported in the 18 months prior to interview.

Example 3: Recall of Stressful Events

- H_0: Events uniformly distributed over time.

  H_0: $p_1 = p_2 = \ldots = p_{18} = 1/18 = 0.055$

  where p_i = probability of event in month i.

  i.e. we would expect about 5.5% of all events per month

Example 3: Recall of Stressful Events Data

month	count	%	month	count	%
1	15	10.2	10	10	6.8
2	11	7.5	11	7	4.8
3	14	9.5	12	9	6.1
4	17	11.5	13	11	7.5
5	5	3.4	14	3	2.0
6	11	7.5	15	6	4.1
7	10	6.8	16	1	0.7
8	4	2.7	17	1	0.7
9	8	5.4	18	4	2.7

Evaluation of Poisson Model

- Let us evaluate the model using Goodness of Fit Statistics
    - Pearson Chi-square test
    - Deviance or Log Likelihood Ratio test for Poisson regression

- Both are **goodness-of-fit test statistics** which compare 2 models, where the larger model is the saturated model (which fits the data perfectly and explains all of the variability).

Pearson and Likelihood Ratio Test Statistics

- In this last example, **if H_0 is true** the expected number of stressful events in month i (in any month) is (equiprobable model)

$$E(y_i) = \boldsymbol{m}_i = 147 * (1/18) = 8.17$$

$$\log(\boldsymbol{m}_i) = \boldsymbol{a} \qquad i = 1, \cdots, C$$

- i.e. we have a model with one parameter $\boldsymbol{a}$

Observed and expected count

Month	Count Obs O_i	Count Exp E_i	Month	Count Obs O_i	Count Exp E_i
1	15	8.17	10	10	8.17
2	11	8.17	11	7	8.17
3	14	8.17	12	9	8.17
4	17	8.17	13	11	8.17
5	5	8.17	14	3	8.17
6	11	8.17	15	6	8.17
7	10	8.17	16	1	8.17
8	4	8.17	17	1	8.17
9	8	8.17	18	4	8.17

Pearson Chi-Squared Test Statistic

- The **Pearson chi-squared test statistic** is the sum of the standardized residuals squared

$$X^2 = \sum_{\text{cells } i} \left[\frac{O_i - E_i}{\sqrt{E_i}} \right]^2$$

$$= \left(\frac{15 - 8.17}{\sqrt{8.17}} \right)^2 + \left(\frac{11 - 8.17}{\sqrt{8.17}} \right)^2 + \ldots + \left(\frac{4 - 8.17}{\sqrt{8.17}} \right)^2 = 45.4$$

Pearson Chi-Squared Test Statistic

- **If H_0 is true**

$$X^2 \sim \chi^2_{df}$$

where df = degrees of freedom
= no. of cells – no. of model parameters
= C - 1

- $X^2 = 45.4$ with 17 df (at 5% significance level the value from the chi-square table is 27.6)

  p-value < 0.001 $\rightarrow$ **reject H_0**.

- **Conclusion:** There is strong evidence that the equiprobable model does not fit the data.

Log Likelihood Ratio Test Statistic for Poisson Regression

- The **Log Likelihood Ratio test statistic** (also called **Deviance** of the Poisson Model) is

$$L^2 = 2 \sum_{\text{cells } i} O_i \log \left[\frac{O_i}{E_i} \right]$$

- This can be used as a measure of the fit of the model (**goodness of fit statistics**)

Log Likelihood Ratio Test

- **If H_0 is true**

$$L^2 \sim \chi^2_{df}$$

where df = degrees of freedom

= no. of cells – no. of model parameters

= C - 1

- $L^2 = 50.8$ with 17 df.

p-value < 0.001 $\rightarrow$ **reject H_0.**

- **Conclusion:** There is strong evidence that the equiprobable model does not fit the data.

Remarks

- X^2 and L^2 are asymptotically equivalent. If they are not similar, this is an indication that the large sample approximation may not hold.

- For fixed df, as n increases the distribution of X^2 usually converges to χ^2_{df} more quickly than L^2. The chi-squared approximation is usually poor when expected cell frequencies are less than 5.