Confirmatory Factor Analysis

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Plan

- Measuring concepts using latent variables
- Exploratory Factor Analysis (EFA)
- Confirmatory Factor Analysis (CFA)
- Fixing the scale of latent variables
- Mean structures
- Formative indicators
- Item parcelling
- Higher-order factors

2 step modeling

- ‘SEM is path analysis with latent variables’
- This as a distinction between:
  - Measurement of constructs
  - Relationships between these constructs
- First step: measure constructs
- Second step: estimate how constructs are related to one another

Step 1: measurement

- All measurements are made with error (random and/or systematic)
- We want to isolate 'true score' component of measured variables: $X = t + e$
- How can we do this?
- Sum items (random error cancels)
- Estimate latent variable model

Exploratory Factor Analysis

- Also called ‘unrestricted’ factor analysis
- Finds factor loadings which best reproduce correlations between observed variables
- n of factors = n of observed variables
- All variables related to all factors

Exploratory Factor Analysis

- Retain <n factors which ‘explain’ satisfactory amount of observed variance

- ‘Meaning’ of factors determined by pattern of loadings

- No unique solution where >1 factor, rotation used to clarify what each factor measures

Example: Intelligence

9 knowledge quiz items

Observed Items	*Factor 1*	*Factor 2*	*Factor 3*
Math 1	.89	.12	.03
Math 2	.73	-.13	.03
Math 3	.75	.09	-.11
Visual-Spatial 1	-.03	.68	.07
Visual-Spatial 2	.13	.74	-.12
Visual-Spatial 3	-.08	.91	.05
Verbal 1	.23	.17	.88
Verbal 2	.18	.03	.73
Verbal 3	-.03	-.11	.70

...Factor 9

Example: Intelligence

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$\longrightarrow$ ...Factor 9

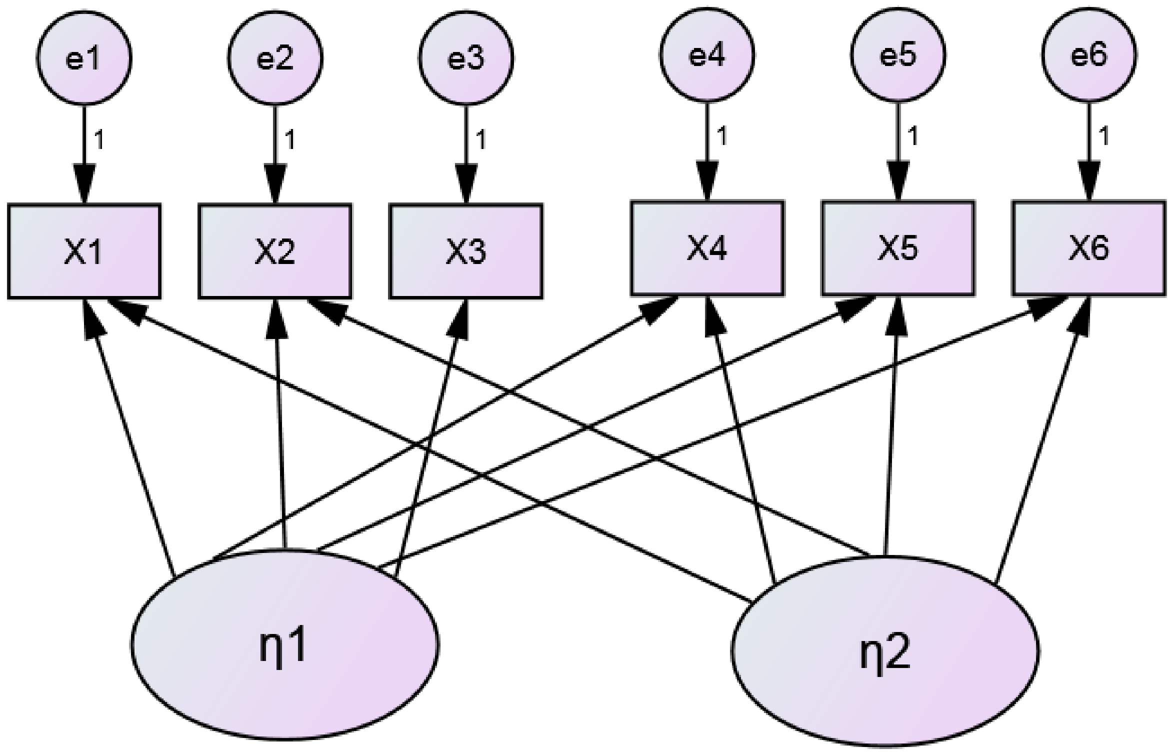
Limitations of EFA

- Inductive, atheoretical (Data->Theory)
- Subjective judgement & heuristic rules
- We usually have a theory about how indicators are related to particular latent variables (Theory-> Data)
- Be explicit and test this measurement theory against sample data

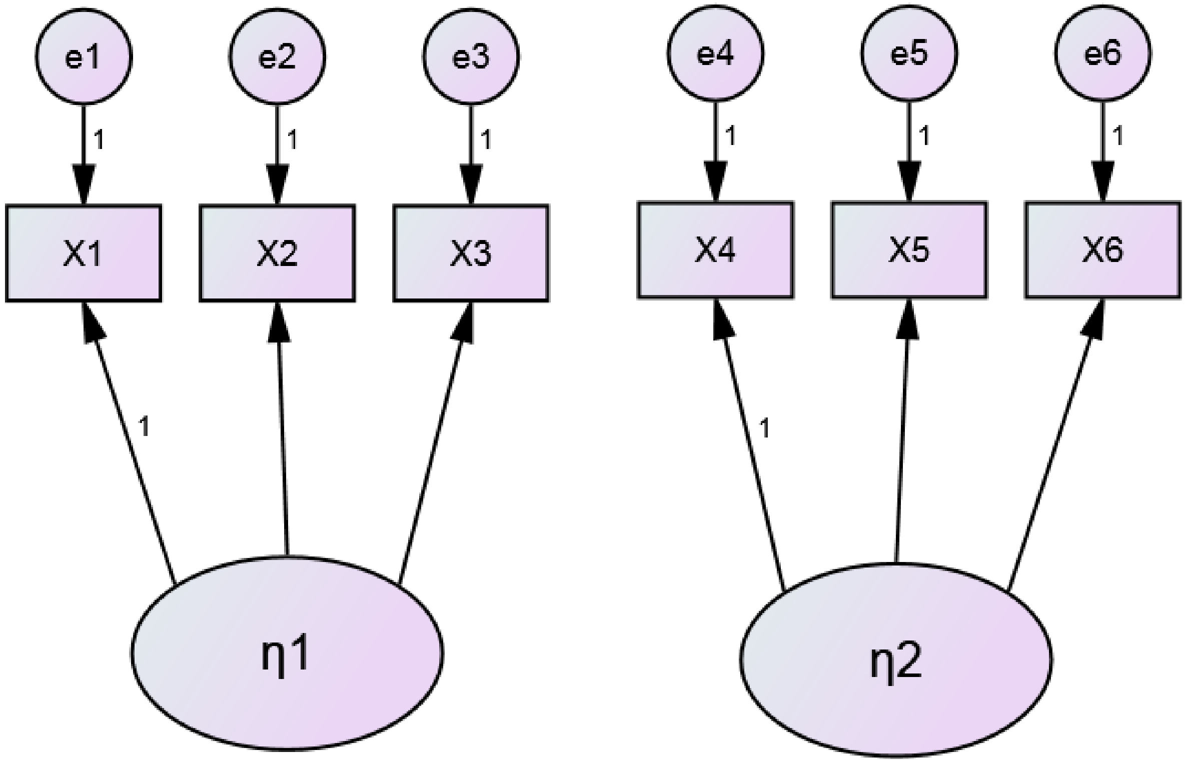
Confirmatory Factor Analysis (CFA)

- Also ‘the restricted factor model’
- Specify the measurement model before looking at the data (the ‘no peeking’ rule!)
- Which indicators measure which factors?
- Which indicators are unrelated to which factors?
- Are the factors correlated or uncorrelated?

Two Factor, Six Item EFA



Two Factor, Six Item CFA



Parameter Constraints

- CFA applies constraints to parameters (hence ‘restricted’ factor model)
- Factor loadings are fixed to zero for indicators that do not measure the factor
- Measurement theory is expressed in the constraints that we place on the model
- Fixing parameters over-identifies the model, can test the fit of our a priori model

Scales of latent variables

- A latent variable has no inherent metric, 2 approaches:
  1. Constrain variance of latent variable to 1
  2. Constrain the factor loading of one item to 1

- (2) makes item the ‘reference item’, other loadings interpreted relative to reference item
  1. yields a standardised solution
  2. generally preferred (more flexible)

Mean Structures

- In conventional SEM, we do not model means of observed or latent variables
- Interest is in relationships between variables (correlations, directional paths)
- Sometimes, we are interested in means of latent variables

  e.g. Differences between groups

  e.g. Changes over time

Identification of latent means

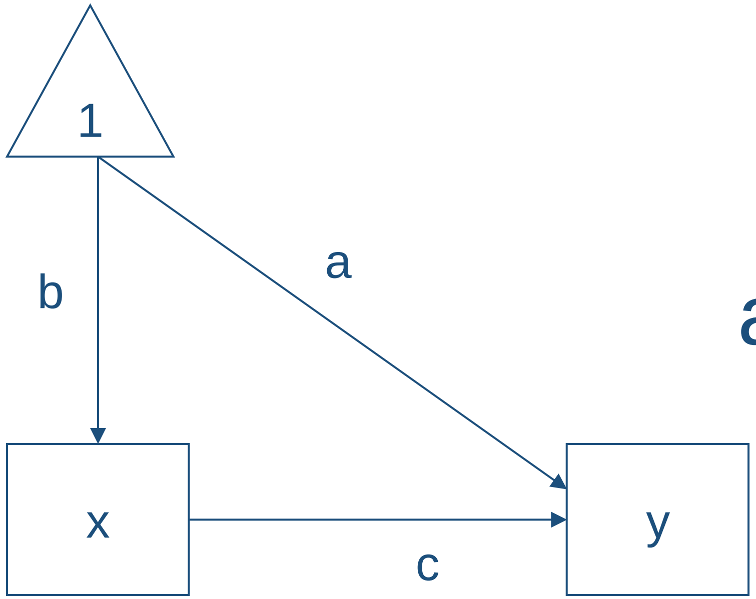
- observed and latent means introduced by adding a constant

- This is a variable set to 1 for all cases

- The regression of a variable on a predictor and a constant, yields the intercept (mean) of that variable in the unstandardised b

- The mean of an observed variable=total effect of a constant on that variable

Mean Structures



b = mean of x

a+(b*c)=mean of y

Means and identification

- Mean structure models require additional identification restrictions
- We are estimating more unknown parameters (the latent means)
- Where we have >1 group, we can fix the latent mean of one group to zero
- Means of remaining groups are estimated as differences from reference group

Formative and Reflective Indicators

- CFA assumes latent variable causes the indicators, arrows point from latent to indicator

- For some concepts this does not make sense

  e.g. using education, occupation and earnings to measure ‘socio-economic status’

- We wouldn’t think that manipulating an individual’s SES would change their education

Formative Indicators

- For these latent variables, we specify the indicators as ‘formative’
- This produces a weighted index of the observed indicators
- Latent variable has no disturbance term
- In the path diagram, the arrows point from indicator to latent variable

Item Parceling

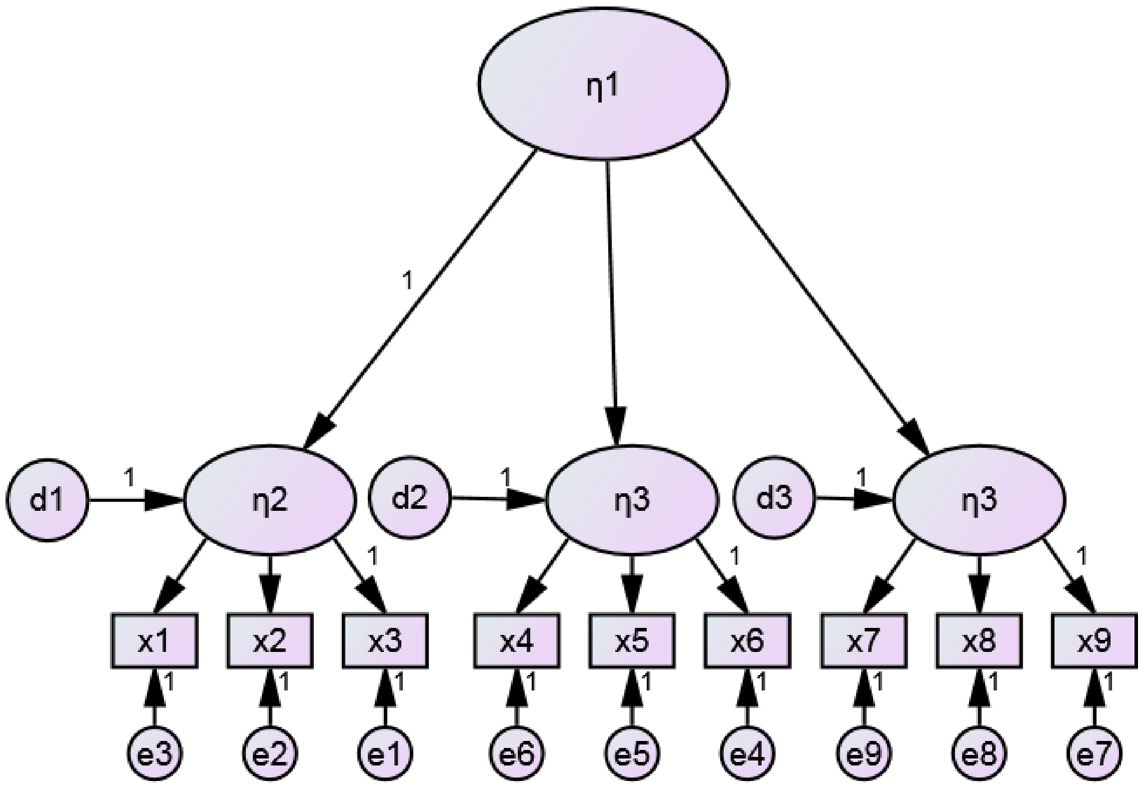
- A researcher may have a very large number of indicators for a latent construct
- Here, model complexity can become a problem for estimation and interpretation
- Items are first combined in ‘parcels’ through summing scores over item sub-groups
- Assumes unidimensionality of items in a parcel

Higher Order Factors

- Usually, latent variables measured via observed indicators
- Can also specify ‘higher order’ latent variables which are measured by other latent variables
- Used to test more theories about the structure of multi-dimensional constructs

  e.g. intelligence, personality

Higher-order Factor Model



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